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NUMERICAL EVALUATION OF COAL PREPARATION DENSITY SEPARATORS PERFORMANCE

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NUMERICAL EVALUATION OF COAL PREPARATION DENSITY SEPARATORS PERFORMANCE

by

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ABSTRACT

The probable error and error area are the most commonly used parameters for performance evaluation of coal preparation density separators. Based on expressing the density separator partition curve mathematically, mathematical expressions for the probable error and error area are derived. The derived expressions offer advantages over existing methods in avoiding the graphical representation of the partition curve and the numerical integration used in the evaluation of the error area. The proposed approach can be easily implemented on digital computers.

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ÉVALUATION NUMÉRIQUE DU RENDEMENT DES SÉPARATEURS DE DENSITÉ LORS DE LA PRÉPARATION DU CHARBON

par

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RÉSUMÉ

L'erreur probable et le champ d'erreurs sont les paramètres les plus souvent utilisés pour faire l'évaluation du rendement des séparateurs de densité lors de la préparation du charbon. La méthode est basée sur l'expression mathématique de la courbe de partition du séparateur de densité, et les expressions mathématiques qui en sont dérivées servent au calcul de l'erreur probable et du champ d'erreurs. L'emploi de ces expressions mathématiques a un avantage sur les méthodes actuelles en éliminant la représentation graphique de la courbe de partition et l'intégration numérique utilisées lors de l'évaluation du champ d'erreurs. La méthode proposée peut être facilement exploitable sur des calculateurs numériques.

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INTRODUCTION

The performance evaluation of density separators used in coal preparation plants are commonly based on the probable error and error area parameters. In a recent article, Tromp introduced a modification of his original method of assessing the performance of coal preparation density separators (1). In his modified method he utilized a graphical approach to evaluate separation efficiency. By introducing a new scaling of the density axis and subsequently the partition number axis he presented the partition curve as two joined straight lines and a similar criterion to the probable error was introduced to evaluate the separation efficiency. The new scaling of the density axis introduced by Tromp was utilized to express the partition curve mathematically and a parameter was introduced as a measure of separation efficiency (2). This numerical method eliminates the need for graphical representation of the partition curve and takes into account the scatter in the partition data.

In the present paper the mathematical expression of the partition curve and the parameter introduced previously are utilized to determine numerically the probable error and error area which are the most commonly used parameters by coal preparation engineers to evaluate the density separation efficiency (2).

MATHEMATICAL DERIVATION

Figure 1 is a representation of "ideal" and "non-ideal" partition curves for a density separator. Starting from the cutpoint $d=d_p$ in Fig. 1, let us construct a new density axis scale to the right and the left of d_p using the relation $[\text{Absolute Value } (d-d_p)]^a$, where "a" is a constant equal to 0.60 (1). To facilitate the mathematical development, we introduce the variable "Δ" as the density deviation from d_p, defined as

$$\Delta = d - d_p \qquad \text{Eq 1}$$

and the variable "pf", probability factor defined as

$$pf(\Delta) = P(d) \qquad d_f \leq d \leq d_p$$

$$pf(\Delta) = 100 - P(d) \qquad d_p \leq d \leq d_s \qquad \text{Eq 2}$$

where d_f and d_s are the float and sink relative densities respectively and P(d) is the partition number or distribution factor at density d. Plotting pf on the new density axis we obtain two differential Gauss probability limbs with their maximum points joined at $d = d_p$ (or $\Delta = 0$) and pf = 50. Therefore, pf can be expressed in terms of Δ as

$$pf = 50\ EXP\ [-\delta_m\ (ABS(\Delta))^{\alpha}] \qquad \text{Eq 3}$$

where $\alpha = 2a$ and δ_m is a constant, equals to δ_{m1} and δ_{m2} for the left and the right limbs respectively and "EXP" and "ABS" are the Exponential and Absolute Value functions respectively. From the definition of the probable error:

$$E_p = (d_{75} - d_{25})/2 \qquad \text{Eq 4}$$

where d_{25} and d_{75} are the relative densities when the partition number is 25.0 and 75.0 respectively, Fig. 1, and using Eq 3 we get

$$E_p = [(d_{75}-d_p)+(d_p-d_{25})]/2 = \Delta_{25}(\delta_{m1}) + \Delta_{25}(\delta_{m2})$$

$$= [(-LN(0.5))^{1/\alpha}/2][(1/(\delta_{m1}{}^{1/\alpha}))+(1/(\delta_{m2}{}^{1/\alpha}))], \qquad \text{Eq 5}$$

where "LN" is the Natural Logarithm.

The error area is defined as the area between the actual partition curve and the step function representing ideal separation, Fig. 1. Using the expression for the probability factor given in Eq 3 we obtain

$$A_e = \int_{d_f}^{d_s} pf(d)d(d)$$

$$A_e = 50\ (\int_0^{\Delta_f} EXP[-\delta_{m1}\Delta^{\alpha}]d(\Delta) + \int_0^{\Delta_s} EXP[-\delta_{m2}\Delta^{\alpha}]d(\Delta)) \qquad \text{Eq 6}$$

where $\Delta_f = d_p - d_f$ and $\Delta_s = d_s - d_p$. Since pf approaches zero for $d \ll d_f$ and $d \gg d_s$ then the upper limits of the two integrations in Eq 5 can be set to infinity. This will facilitate the evaluation of these two integrations in a closed form. Using integrals tables, it can be shown that:

$$A_e = 50\ \Gamma(1+1/\alpha)[(1/(\delta_{m1}^{1/\alpha})) + (1/(\delta_{m2}^{1/\alpha}))] \qquad \text{Eq 7}$$

where "$\Gamma(.)$" is the Gamma Function and its tables are known (3). Equations 4 and 6 express E_p and A_e in terms of δ_{m1}, δ_{m2} and α. Also it is important to note that

$$A_e/E_p = 100\ \Gamma(1+1/\alpha)/(-LN(0.5))^{1/\alpha} = \text{constant} \qquad \text{Eq 8}$$

which indicates that the relationship between A_e and E_p is linear. The effect of variation of α = 2a (a=0.5 for approximately 40 mm particle size and a=0.67 for approximately 0.5 mm particle size) is shown in Fig. 2. Which indicates that the ratio A_e/E_p becomes large as α decreases.

NUMERICAL VERIFICATION

For the sake of continuity, verification will be performed with the same washability data used by Tromp and the author and other (1,2). For the jig washer (Table 1), the results reported previously were a=0.6, α=1.2,

Table 1 - Partition data calculations for fine coal jig washer (4)

Density stages	Mid density $[d]_m$	Feed coal	Washed coal in % of feed	Sinks in % of feed	Distribution factor [df]	Probability factor [pf]	δ	δ_m
-1.35	1.325	52.450	52.210	0.236	0.45	0.45	6.706	
1.35-1.40	1.375	26.460	26.280	0.180	0.68	0.68	6.650	
1.4 -1.45	1.425	6.230	6.160	0.065	1.05	1.05	6.539	
1.45-1.50	1.475	2.340	2.307	0.033	1.41	1.41	6.654	
1.5 -1.55	1.525	1.975	1.933	0.042	2.12	2.12	6.548	
1.55-1.60	1.575	1.154	1.120	0.034	2.95	2.95	6.580	6.496
1.60-1.65	1.625	0.955	0.914	0.041	4.30	4.30	6.482	
1.65-1.75	1.700	1.570	1.452	0.118	7.50	7.50	6.255	
1.75-1.85	1.800	1.252	1.079	0.173	13.80	13.80	6.195	
1.85-2.00	1.925	1.263	0.924	0.339	26.90	26.90	6.290	
2.00-2.10	2.050	0.700	0.371	0.329	47.00	47.00	6.762	d_p=2.07
2.10-2.20	2.150	0.644	0.266	0.378	58.60	41.40	3.910	
2.20-2.30	2.250	0.683	0.211	0.472	69.00	31.00	3.742	
2.30-2.40	2.350	0.620	0.148	0.472	76.00	24.00	3.381	3.693
2.40-2.50	2.450	0.533	0.035	0.498	84.50	15.50	3.740	
+2.50		1.168	0.140	1.028	-	-	-	
TOTAL		99.997	95.55	4.438				

Table 2 - Partition data calculations for heavy medium separator (4)

Density stages	Mid density $[d]_m$	Feed coal	Washed coal in % of feed	Sinks in % of feed	Distribution factor [df]	Probability factor [pf]	δ	δ_m
-1.35	1.325	34.49	34.47	0.02	0.052	0.052	223.064	224.71
1.35-1.375	1.3625	29.52	26.99	2.53	8.57	8.57	226.356	d_p=1.38
1.375-1.40	1.3875	12.70	4.64	8.06	63.46	36.54	111.256*	94.344
1.40-1.50	1.45	14.02	0.29	13.73	97.93	2.07	77.412*	
+1.50		9.27	Nil	9.27	-	-	-	
TOTAL		100.00	66.39	33.61				

* δ values are not close possibly due to error in approximation of density mid point.

δ_{m1}=6.496 and δ_{m2}=3.693 (2). Using Eqs 5 and 7 for E_p and A_e respectively we obtain

$$E_p = 0.0775 + 0.1240 = 0.2015$$
$$A_e = 9.89 + 15.83 = 25.72$$

which indicates that E_p agrees with the result reported by Tromp and the value of A_e corresponds to a conventional error area of 128.5 (1). In this case the ratio A_e/E_p is :

$$A_e/E_p = 127.64.$$

For the heavy medium cyclone washer (Table 2), the results reported previously were a=0.6, α=1.2, δ_{m1}=224.71 and δ_{m2}=94.344 (2). From Eqs 5 and 7 respectively we obtain

$$E_p = 0.00404 + 0.00833 = 0.0124$$
$$A_e = 0.516 + 1.064 = 1.58$$

which again indicates that E_p agrees with the result obtained by Tromp and A_e corresponds to a conventional error area of 7.90 (1). In the case of heavy medium cyclone washer the ratio A_e/E_p is:

$$A_e/E_p = 127.71$$

it is noted that this A_e/E_p ratio is in close agreement with that obtained for the jig washer.

CONCLUSIONS

Numerical methods are developed to determine the probable error and error area used in the performance evaluation of density separators. A linear

relationship between the probable error and error area was established. The numerical methods presented are easy to implement on digital computer and provide the coal preparation engineer with quick evaluation of separation efficiency. These numerical methods are useful in generating numerical models for density separators.

REFERENCES

1. Tromp, K.F. "Modified methods of assessing separation efficiency", Mine and Quarry 10: 6: 49-50: 1981.

2. Salama, A.I.A. and Mikhail, M.W. "A method for evaluating density separation efficiency"; Mine and Quarry 11: 12: 28-30: 1982.

3 Beyer, W.H. "CRC Standard Mathematical Tables", 26 th Edition, CRC Press, Inc.; Boca Raton, Florida, No. 662, 1981.

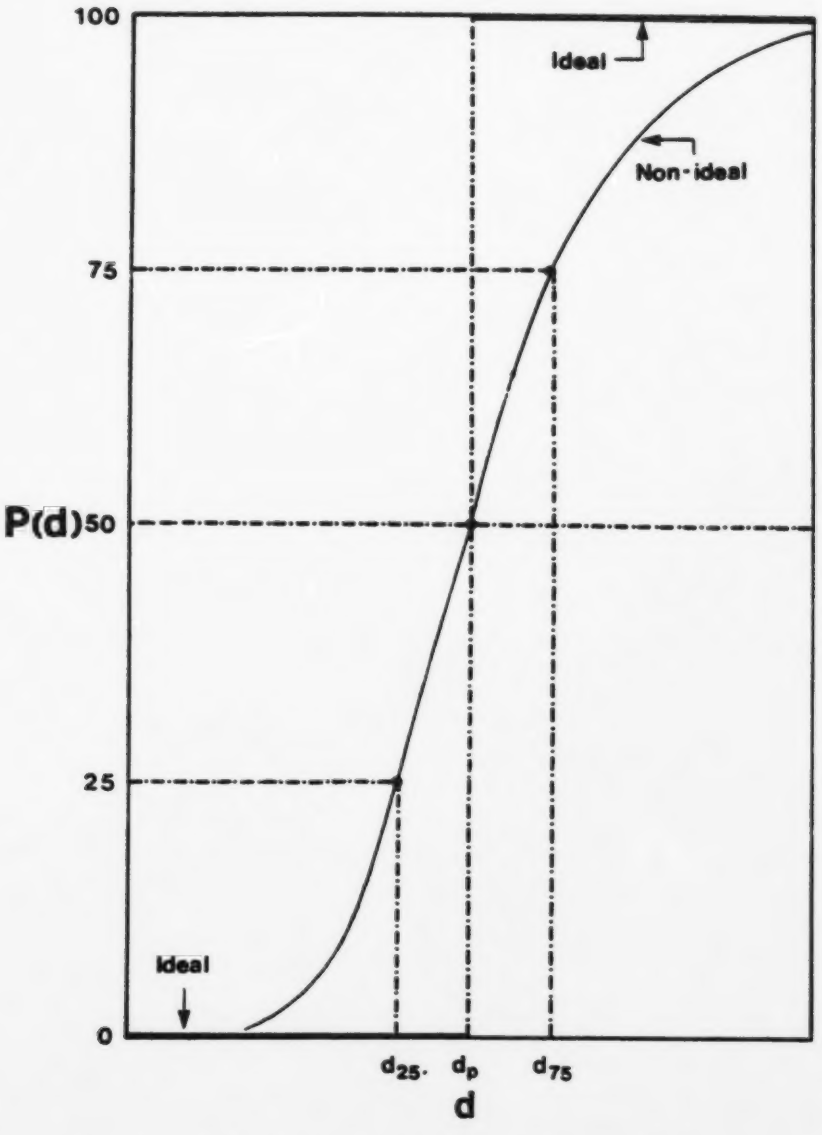

Fig. 1 - Ideal and non-ideal partition curves

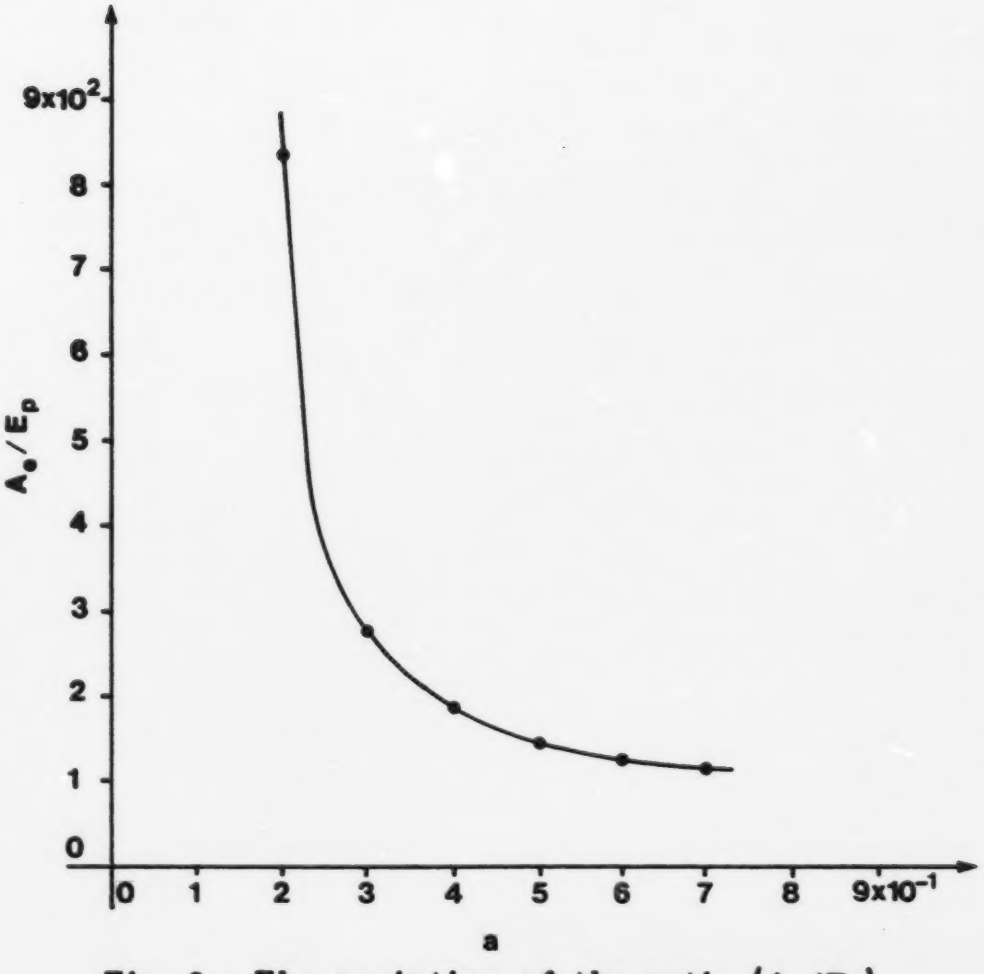

Fig. 2 - The variation of the ratio (A_e/E_p) as function of "a"